

*Velociraptor mongoliensis*  
vs. *Thescelosaurus neglectus* : Winning  
Strategies and Genetic Algorithms

Team Number 46 for the Mathematical Contest in Modeling

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# 1 Restatement of the Problem

During the Cretaceous period (65 to 144 million years ago), eastern Asia and western North America were joined together into one large land mass. Among its inhabitants was an infraorder of predatory dinosaurs called Deinonychosauria, meaning “terrible clawed lizards,” so named for the large, sickle shaped claws on their feet. They were relatively small, 1.5 to 4 meters long from head to tail, bipedal with grasping hands, swift, and deadly. There is a distinct possibility that they were intelligent enough to have hunted in packs. Among them, the velociraptor, *Velociraptor mongoliensis*, is of interest to us.

Also at this time, a number of herbivores were evolving, including the family Hypsilophodontidae, so named for the ridges on their teeth. Several *Hypsilophodon* fossils were found together at one important dig, indicating that they may have lived in herds. They were bipedal and agile, possibly living a lifestyle similar that of gazelles or deer, and using their speed to escape predators.

One member of this family was *Thescelosaurus neglectus*, which inhabited what would later become Canada. It was larger and less agile than its relatives, and therefore a likely target for predators.

The subject of this project is to model the possible strategies velociraptors might have used while preying on thescelosaurus, and the related defense strategies used by the prey. Paleontologists will use these results to compare the velociraptor and thescelosaurus to modern predators and their prey, such as wolves and deer, and cheetahs and gazelles, in the hope of better understanding their lifestyle. In the first part, the raptor is assumed to be acting alone against a single thescelosaurus. The second part considers a pair of raptors and their possible group strategies.

## 2 Assumptions

The following assumptions are intended to simplify the problem for initial analysis while still retaining its most meaningful aspects.

- The velociraptor is 3 m long and weighs 45 kg. This estimate, given in the original statement of the problem, is contradicted by other sources which indicates that it is not clearly known. However, the raptor's exact size is not central to the mathematical model and a rough estimate of 2-3 m suffices.
- The thescelosaur is approximately the same size as the raptor. Again, this estimate is debatable, but the exact size is not critical.
- If the raptor can get to within arm's reach of the thescelosaur, he can kill it. This assumption is based on [2, page 113]. It describes a fossil site where the skeletons indicate that a velociraptor pounced on another dinosaur, grabbed part of its head, and cut its underside open with the claws on his feet. This suggests that the general hunting strategy was chase, grab, and slash.
- Both dinosaurs can be represented as points. Their shape is partially taken into account by the concept of the predator's arm reach.
- The velociraptor can run at top speed (60 km/h) for fifteen seconds, then must rest. He does not have the option of slowing down once he has reached this speed and must use the 15 seconds all at once.
- The thescelosaur can run at top speed (50 km/h) indefinitely. However, his top speed is significantly slower than the raptor's top speed.
- The velociraptor cannot maintain a speed comparable to its prey for very long and will lose distance if he runs at a comfortable speed. The chase must therefore be rather short.
- Both the velociraptor and the thescelosaur run at top (linear) velocity throughout the chase. This means that the chase is limited to fifteen seconds, because after that the raptor has to stop and rest. Since he can't keep up with the thescelosaur in a prolonged chase, there appears to be no advantage for the raptor in running at a comfortable speed which loses distance.

In fact, in any chase, the velociraptor must eventually kick into high gear to have any hope of catching his prey, and the rest of the fight reduces to the simplified case described here.

- As a consequence of the raptor's limited endurance, if he does not catch the prey during his 15 seconds of high speed running, the thescelosaur is assumed to escape.
- Both dinosaurs have a minimum turning radii which can be used to compute their maximum angular velocities. When they turn, they are agile enough to instantaneously assume this angular velocity. The velociraptor at top speed can run in arc with maximum radius of curvature 1.5 m, which amounts to 11.1 rad/s. For the thescelosaur, the turning radius is 0.5 m and the angular velocity is 27.8 rad/s.
- Both dinosaurs have instant reaction time. That is, if one of them changes direction, the other perceives it immediately. Part of our model involves a simulation, and either dinosaur may take action in as quickly as one time step.
- Initially, the thescelosaur notices the velociraptor when he is between 50 and 15 meters away, depending upon visibility and the alertness of the thescelosaur. Later analysis will show that if the raptor is discovered at more than 40 meters away, the thescelosaur will escape. Therefore, most of the model focuses on what happens if the raptor can sneak in fairly close.
- Once the chase has begun, both dinosaurs know exactly where the other is, how fast he is going, and in what direction.
- Both are also aware of how long the raptor can maintain his top speed, and how much time he has left.
- The chase takes place on a flat, featureless plain. There must be enough vegetation to feed the thescelosaur, so the setting of the hunt must be either a forest, a swamp, or a plain. The forest and swamp are full of obstructions which add too much complexity to the model, so those cases are not considered.
- Both dinosaurs are as intelligent as necessary, within reason. They are assumed not to have tool-making abilities or technology, but any other

strategy is possible. In particular, the velociraptor is capable of stealth and can sneak in fairly close to his prey.

### 3 Analysis of the Problem

We shall begin by considering the behavior of some simple strategies for a single velociraptor and a thescelosaur.

#### 3.1 The Linear Case

As a simplest case, what happens when we only allow motion in a straight line? Naturally, the velociraptor will run directly towards the thescelosaur, who will be running directly away. Thus the velociraptor can close the initial distance by

$$\text{velociraptor running time} \cdot (\text{velociraptor speed} - \text{thescelosaur speed})$$

in the time before its muscles give out. Under the assumed values of these variables, this is 40 meters. Remarkably, this means that under good conditions the thescelosaur can perceive the velociraptor far enough away (up to 50 meters) so that it can be assured of escape.

It seems likely that the velociraptor can't always afford to wait for a low-visibility day to eat, and so it follows that the velociraptor *must* be intelligent enough to use stealth or to hunt in groups in order to catch its prey. Even leaving that aside, this result makes good evolutionary sense. For it means that the velociraptor will mostly catch those thescelosaurus in the herd which have diminished perception or diminished running speed, namely the old and the infirm. This prevents the velociraptors from killing off their food supply; it may even help the herd to prosper.

#### 3.2 Thescelosaur Escape Strategies

If, however, a thescelosaur is caught in non-optimal perception conditions, running directly away isn't enough. In this case the velociraptor has the advantage. We consider a velociraptor using the straightforward strategy of always moving directly towards its target. This could easily be improved—for instance by enabling the raptor to aim ahead of a thescelosaur moving at an angle—but for the moment the simple strategy will be more than good enough. So if even this relatively unintelligent velociraptor manages to get close to its prey before being seen (say, within 20 meters), what can the thescelosaur do to escape?

To explore this situation a simple simulator was constructed (see Appendix A for details).

It is fairly evident that the thescelosaur's hope lies in taking advantage of the velociraptor's large turning radius. Because its turning radius is so much smaller than the velociraptor's, it will perhaps be able to maneuver in such a way that the velociraptor is unable to close the distance as swiftly.

An initial attempt examines the thescelosaur strategy of always keeping itself moving in a direction at an angle to the velociraptor's motion (see Figure 1).

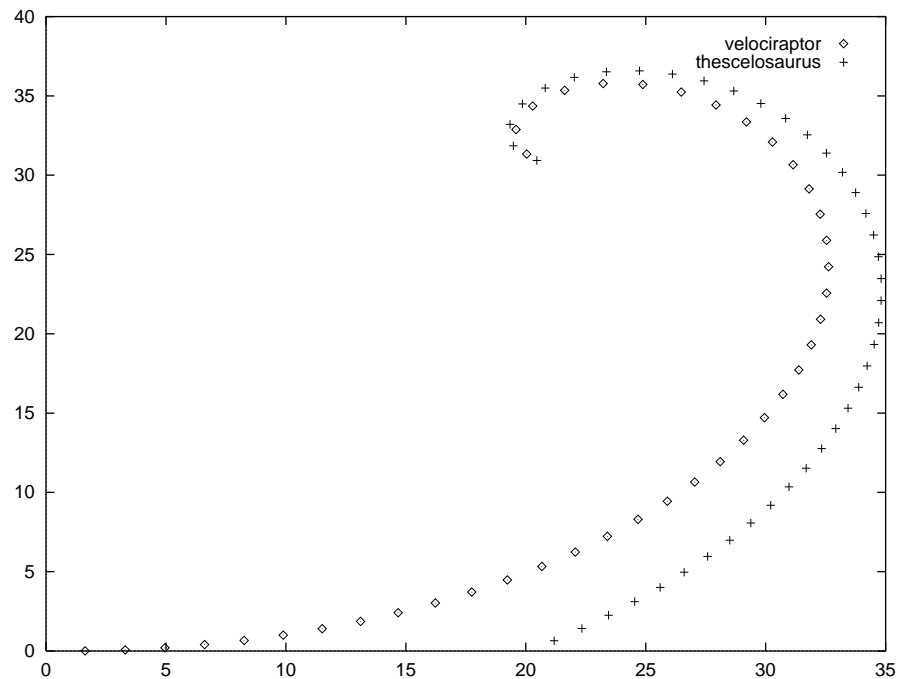


Figure 1: Maintaining angle strategy.

Unfortunately, starting at 20 m the velociraptor has enough time to react to the thescelosaur's turning, and so never has to push the bound on how sharply it can turn. In fact the thescelosaur is caught more quickly using this strategy than when it simply runs directly away.

To take better advantage of the velociraptor's limited turning, it seems that the thescelosaur needs to turn when the velociraptor is *closer*. This will force the velociraptor to make a much sharper turn to move towards its prey. The velociraptor will then overshoot and have to curve back in order to maintain a bead on the thescelosaur.

The most significant gains are made by the thescelosaur when it actually follows a *loopback* strategy, curving sharply around the imminently approaching hunter. The velociraptor will then be forced to turn all the way around in order to follow the thescelosaur, who has gained a significant amount of time (see Figure 2).

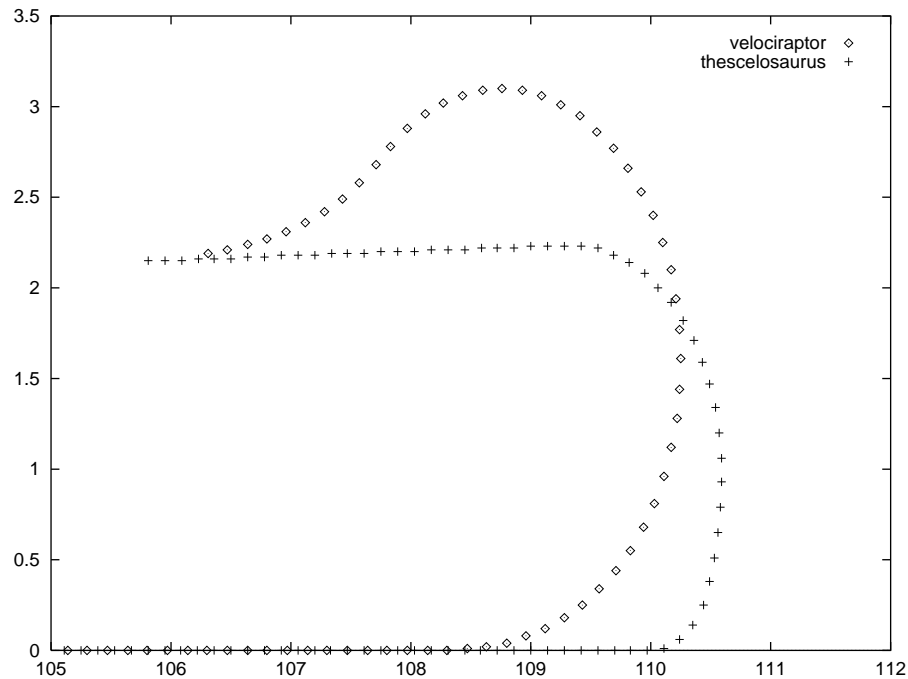


Figure 2: Loopback strategy.

There is a serious flaw with this strategy. Although the thescelosaur gains distance while the velociraptor is executing the upper half of its turn, to do so it has to come as close as a half a meter to the raptor, running perpendicularly in front of the raptor. So it is quite possible that the raptor could make a sudden lunge and bring a quick end to this strategy. Moreover,



since the simulation is representing meters-long creatures by points, it seems even more likely that coming so close would mean death.

A good way to imagine this process is as follows. The thescelosaur will run directly away from the velociraptor as long as possible. Then, just as the raptor is about to leap to the kill, the thescelosaur swerves sharply around its hunter. At this point there is a chance that the velociraptor will be able to lunge and wound the thescelosaur enough to stop its flight. On the other hand there is a certain probability that the raptor will miss because of the extreme speed of the chase. Moreover, it should be remarked that *Thescelosaurus* developed protective bony studs in its back, perhaps for just this kind of encounter ([2, page 141]). Nonetheless, the thescelosaur has significant chance of death using this strategy.

On the other hand, this loopback strategy was probably the thescelosaur's only chance for life. It is clear that the thescelosaur needs to use its more agile turning to evade its predator. However, if it turns while the velociraptor is still at a significant distance, the velociraptor—even one using the “dumb” directional strategy outlined above—is more than able to compensate. Also, if it tries to turn and still continue forward—for instance, in a forward zigzag motion—our simulation indicates that not nearly enough time is gained, and the velociraptor will almost certainly be able to close before its muscles fatigue. A loopback seems to be absolutely necessary.

It should be noted that this maneuver can be iterated to prolong the chase until the velociraptor gives up; however, with each swerve the thescelosaur runs a large chance of cutting it just too close. The iterated pattern can take on any number of interesting shapes (see Figures 3, 4, 5), depending on the thescelosaur's precise strategy for how close it allows the velociraptor and how sharply it turns; the basic idea is the same for all.

We feel that it is unlikely that these kinds of maneuvers serve as anything more than a last-ditch effort. It is conjectured that *Thescelosaurus* was a herd animal, and as such survival would be guaranteed by the fact that the old and infirm would be killed first, not quite so much by the ability of any individual to evade the determined hunter.

### 3.3 Velociraptor Hunting Strategies

So far we have concentrated on the optimal strategy for the thescelosaur, assuming that the velociraptor is using the straightforward strategy of always running directly toward its prey. The question next arises: how much is it

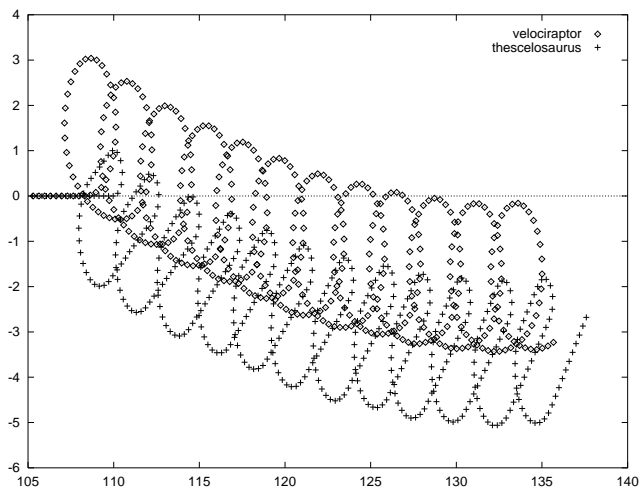


Figure 3: Zigzag loopback. That the thescolosaurus seems to run toward the velociraptor at points is an illusion caused by the scale and lack of time reference.

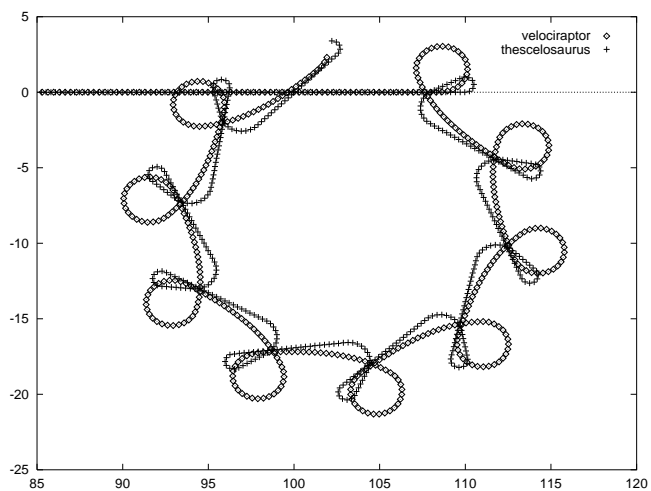


Figure 4: Dance of death.

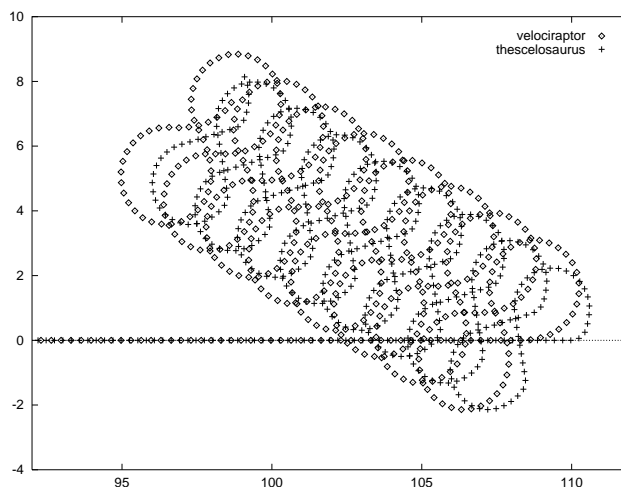


Figure 5: Backwards figure-8 loopbacks.

possible to improve on this hunting strategy for a single hunter?

Our conjecture is: not much. It has been seen that under good conditions for the thescelosaurus's perception, a single velociraptor will be unable to approach closely enough to catch the other dinosaur; on the other hand, when the velociraptor is stealthy enough to approach more closely, the thescelosaurus is unlikely to escape. (It can only escape even the “dumb” strategy through a series of swerves, each of which the thescelosaurus will be lucky to survive.) In other words, whether the single velociraptor catches its prey seems to depend almost entirely on how closely it can approach before beginning its short sprint.

We certainly imagine that the intelligent hunter would follow a slightly better strategy, one that would anticipate its prey's location when the prey is running at an angle. That is, instead of always aiming directly at its prey, it would aim slightly ahead. It seems, though, that this strategy would at best provide a marginally better chance for the velociraptor to take down the thescelosaurus when it swerves—a chance which is already quite high.

This analysis certainly makes sense considering what is conjectured about the hunting behavior of *Velociraptor*. Firstly, it was hunting a herd animal, so there would be the opportunity for solo hunters to pick off the weak and the stragglers. However, it certainly would have needed the ability to kill

stronger members of the herd when it was absolutely necessary. To this end, *Velociraptor* seems to have organized in hunting packs, and would have more likely hunted in pairs. This naturally leads to the consideration of optimal hunting strategy for a velociraptor hunting pair.

Before advancing to that particular question, we would like to discuss an entirely different approach to solving this problem. So far all we have really done is examine what the possible hunting and evasion strategies are, and use a simulator to see which are most effective. However, there is no guarantee that our consideration of various strategies has been exhaustive.

Biology seems full of examples of animal instincts which lead to behavior which is very effective, but which no human would ever consider. In particular, we are not hunters or prey, and so our perspective on this situation is totally foreign. We need some approach to the problem which will provide possibilities which we would never think of on our own. A natural way to explore the various possibilities is to use the same process as life itself.

### 3.4 Using a Genetic Algorithm

While it is fairly easy to model any given strategy for the predator and the prey, coming up with those strategies is not. The revised model is not concerned with any single strategy, but rather with the natural forces that would have caused those strategies to evolve in living dinosaurs.

Each creature in the physical simulation has a “brain” which tells it what direction to go next. So far, only a handful of hard-wired brains have been tried. Consider a more flexible brain which can take information about both dinosaurs and output one of the actions “turn left,” “turn right,” or “go straight.” This process is repeated for each time step of the physical simulation. Since both animals are assumed to be going at top speed, there is no need to include “slow down” or “speed up.” Since angular acceleration is instantaneous, there is also no need specify how far to turn. Smooth turns are made by intermixing a bunch of “turn” and “straight” instructions, and sharp turns are made by using more “turns.” Keep in mind that the “brain” here an abstract combination of the physical brain, reflexes, instincts, and so on of a real dinosaur.

In nature, the dinosaur’s instincts, reasoning, and reflexes are all combined in the decision making process. These factors are encoded in their genes and are passed on from one generation to the next. A dinosaur with a “good” brain is most likely to reproduce. A “dumb” predator on the other hand is likely to die of starvation, and a “dumb” prey will probably get caught and eaten. Under these circumstances, natural selection takes over, and as the best members of a species reproduce and recombine their genes, successively better dinosaurs evolve.

It is possible to simulate evolution with a computer program. Each brain is represented as a bunch of bits (0s and 1s) which take several bits of input, and produces one of the three strategies “left,” “right,” or “straight.” The physical simulator described in the first part of this section determines which brains are better than others and may be used to sort the population in order of fitness. The best brains reproduce and the worst brains die and get replaced. Starting with a population of randomly created brains, the population will eventually home in on the optimum strategies. The details of this *genetic algorithm* are in **THE APPENDIX**.

This simple model has the ability to create any possible strategy that either dinosaur might use, even ones that are counterintuitive to humans and would not otherwise be discovered.

The great advantage to genetic algorithms is their speed. There are  $2^{1024}$  possible brains in the implementation used in this project. Searching all of them exhaustively for the best ones for each species would take on the order of  $10^{300}$  years, which is many orders of magnitude larger than the age of the universe. On the other hand, we estimate the running time of our genetic algorithm combined with the physical simulation to take on the order of a few years to run on a Sparc 5 computer. That's still too long to get results for this paper, but much better than the alternative.

The algorithm can be sped up by decreasing its precision. Currently, each time tick in the physical simulation is 5 ms, and could be made larger by a factor of ten. That sacrifice of accuracy brings the running time down to a matter of months on a Sparc 5, although the chance of getting meaningful results is difficult to predict. It can also be computed in parallel, with each processor of a super computer performing the work for a handful of dinosaurs. In this case, the algorithm could compute thousands of generations in a matter of hours. Therefore, despite the fact that this simulation cannot be run in time to give results for this paper, it is still a reasonable computation.

The disk included with this paper contains the code for the physical simulation, the genetic algorithm, a sample problem for the genetic algorithm which finishes running in just a few hours, and the combined evolution engine and dinosaur simulation.

### 3.5 Hunting Strategies for a Velociraptor Pair

We would like to note that the genetic algorithm model can easily be extended to more complicated hunting situations. In particular, one can begin with velociraptor and thescelosaur genomes that have already evolved powerful one-on-one strategies, and then put them in a two-on-one situation. This will force the genomes to adapt. By starting with already highly evolved genomes, one can save a significant amount of computer time.

We recommend the genetic algorithm approach because of its robustness. It will discover effective strategies that might be overlooked. However, we will also consider here some particular strategies for a hunting pair that we have devised.

When exploring the dynamic between a single velociraptor and a thescelosaur, we concentrated on thescelosaur strategies for evasion when the velociraptor had managed a close approach; for, with such a close approach, it seemed the velociraptor was almost assured of catching its prey. Moreover, when the velociraptor was farther away, the thescelosaur was assured of escape.

When the velociraptors hunt in pairs, the situation changes. Since the likelihood of a kill given a close approach is already high for one predator, we shall concentrate on strategies for the velociraptor pair when they are detected at a greater distance; in fact, near the upper limit of thescelosaur perception (50 meters).

#### 3.5.1 Naive Strategy

The simple velociraptor strategy we have already seen is quite effective if the velociraptor begins close to its prey. In this section, we consider the effectiveness of this strategy from a greater distance with two hunters.

The raptors in Figure 6 begin at significant distance from their prey on opposite sides of it. They then proceed directly towards it. The thescelosaur takes its simple evasion strategy of running directly away from the closest raptor.

In a one raptor scenario this case would clearly end with a safe getaway for the thescelosaur. It is outside the 40m safe range in a straight chase; however, the thescelosaur quickly discovers that it cannot run straight away from either raptor (without entering the arms of the other), and in compromising on a new path to keep away from both raptors, it loses significant distance. The