A stochastic model of language change through prediction-driven instability

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X
Language variation

- Multiple ways to say the same thing
  Ex: Late Middle English
  (1) I know not the muffin man
  (2) I do not know the muffin man

- Can be correlated to...
  - Location
  - Context
  - Gender
  - Attitude
Language change

Goal: Model spontaneous internally-driven language change

- Grows from variation
- Each speaker uses a mixture of old & new
- One variant replaces the other, mostly monotonically
Ex: *do*-support

Verb raising

\[
\text{IP} \quad \text{IP} \quad \text{I'} \quad \text{I} \quad \text{I} \quad \text{VP} \\
\text{DP} \quad \text{I} \quad \text{I} \quad \text{t} \quad \text{t} \\
\text{[\/*V,pres\]-know} \\
\]
Affix hopping

Diagram:

- IP
- DP
- I
- [V,pres]
- VP
- t
- V
- know
Mean field differential equation

- One parameter $\Rightarrow$ 2 grammars
  - $G_1$: param unset
  - $G_2$: param set

- $m =$ mean rate of $G_2$

- $g(m) =$ mean rate of children

Birth/death process

$$\frac{dm}{dt} = g(m) - m$$
Unstructured Markov chain

Time step $t$

Each agent lives w/ some prob

Time step $t+1$

Dies w/ some prob, & replace w/ child

Dist of children for time $t+1$

= fn of state at time $t$
Time trace of one trajectory

Mean usage rate of $G_2$ as a function of time
Age structured Markov chain: Incrementation

At time step $t$, individuals can age, die, or give birth. At time step $t+1$, the population size is incremented by the number of births. The distribution of children for time $t+1$ is a function of the state at time $t$.

mean

NEW (prediction)

Dist of children for time $t+1$

= fn of state at time $t$
Time trace of one trajectory

Mean usage rate of $G_2$ among young group as a function of time
Time trace of one trajectory: One transition

Mean usage rate of $G_2$ among young group as a function of time
Young distribution and old distribution

Number of people with usage rate $z$ (vertical axes). Time increases left to right. Top: Old. Bottom: Young. Darker $\Rightarrow$ more.
Old mean rate vs. young mean rate
Mean field differential equation w/ age structure

- $v = \text{mean usage rate of } G_2 \text{ in the young group}$
- $w = \text{mean usage rate of } G_2 \text{ in the old group}$
- Birth, learning, aging, death

$$\frac{dw}{dt} = v - w$$
$$\frac{dv}{dt} = g(r(v, w)) - v$$

- $r(v, w) = \text{prediction}$
Mean field differential equation w/ age structure

- \( v = \text{mean usage rate of } G_2 \) in the young group
- \( w = \text{mean usage rate of } G_2 \) in the old group
- Birth, learning, aging, death:
  - \( \frac{dw}{dt} = v - w \)
  - \( \frac{dv}{dt} = g(r(v, w)) - v \)
- \( r(v, w) = \text{prediction} \)
Learning two independent parameters

\[ m_2 = \frac{31}{32} \]

\[ m_2 = \frac{1}{32} \]

- Parameters 1 & 2
- Means \( m_1 \) & \( m_2 \)
- Distribution of child speech = function of \( m_1 \) and \( m_2 \)
- Darker = more
Two independent parameters: Time trace

Mean usage rates among young group as a function of time
Learning two dependent parameters

\[ m_2 = \frac{31}{32} \]

\[ m_2 = \frac{1}{32} \]

- Parameters 1 & 2
- Can only set 2 if 1 is set
- Means \( m_1 \) & \( m_2 \)
- Distribution of child speech = function of \( m_1 \) and \( m_2 \)
- Darker = more

\[ m_1 = \frac{1}{32} \]

\[ m_1 = \frac{31}{32} \]
Two dependent parameters: Time trace

Mean usage rates among young group as a function of time
Conclusion

- Model of variable speech — usage rates
- In unstructured population
  language doesn’t change spontaneously
- In structured population
  language can change spontaneously
- Prediction-driven instability
Bibliography


Prediction function

\[ y = \frac{1}{1 + \exp(k \, t)} \]
More *do*-support data

**Do Support Frequencies**

- Frequency of *Do*
- Date

Legend:
- Trans Affirm Q
- Intrans Affirm Q
- Neg Decl
- Affirm Obj Q
- Neg Q