Truths and Consequences Sprint

All-Day Sprint
Math Meet 2007
College of Charleston

Part I: Each of the numbered statements below is either true or false. Put a check in the box next to those statements that are *true*.

- ☐ 0. This statement is true and the next two are false.
- ☐ 1. This statement and the next are both false.
- ☐ 2. The next statement is true.
- ☐ 3. Exactly four of these statements are true.
- ☐ 4. All of these statements are true.
- ☐ 5. There is a unique x such that statements x and $x+1$ are both false.
- ☐ 6. The next two statements are either both true or both false.
- ☐ 7. There is a unique x such that statements x and $x+1$ are both true.
- ☐ 8. Statement 9 is true.
- ☐ 9. A non-empty complete metric space is never a union of a countable number of nowhere dense sets.

Part II: Let D be the set consisting of the numbers of statements from Part I that are true. Let S be the set of whole numbers whose base 10 representation uses each element of D exactly once. For our purposes, an initial 0 is okay. So for example, if you conclude that statements 0, 2, and 3 are true and the rest are false, then $S = \{023, 032, 203, 230, 302, 320\}$. The statements that follow are either true or false. The set R is the largest subset of S consistent with the truth values of these statements. Your job is to put a check in the box next to those statements that are *true* and find all elements of R.

- ☐ A. Statement B is true and this statement and statement J are either both true or both false.
- ☐ B. At least two of statements C, D, and F are false.
- ☐ C. The number of elements of R is odd.
- ☐ D. This statement and statement B are both false.
- ☐ E. Every pair of elements of R differ by a multiple of 9.
- ☐ F. Let $r(n)$ be the number obtained by reversing the digits of r excluding any initial 0. For each $n \in R, r(n) \in R$.
- ☐ G. There is exactly one prime number that divides evenly into all the elements of R.
- ☐ H. R is the largest subset of S such that statement G is true.
- ☐ I. All elements of R are prime and statement F is true.
- ☐ J. All false statements in Part II occur in consecutive pairs and there are never three or more consecutive false statements.

$$R = \Big\{ \qquad \Big\}$$