

The Runge-Kutta method

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In this project, you will implement and use the Runge-Kutta fourth order numerical method. (Note that “Runge” is a German name and is pronounced like “roonga” and doesn’t rhyme with “grunge.”)

Part I.

Following section 2.6 in Edwards and Penney, implement the Runge-Kutta method for a first-order scalar equation in your favorite language (Maple, Mathematica, Matlab, etc.). Use it to compute π to within a difference of less than 5×10^{-10} . That will give you nine decimal places of accuracy.

Part II.

Following section 4.3 in Edwards and Penney, implement the Runge-Kutta method for a first-order system. (Be aware that Maple sometimes requires special syntax for matrix and vector arithmetic that is different from the syntax for scalar arithmetic. Mathematica is more consistent about its notation.) Check that your implementation is correct by reproducing the tables on p. 271.

Using your program (not a numerical method built into Maple!), plot several solutions in the (x, y) plane to the following system for a variety of positive values of a and b :

$$x' = 1 - (b + 1)x + ax^2y \tag{1}$$

$$y' = bx - ax^2y \tag{2}$$

(This is a model of an oscillating chemical reaction.) Determine approximately which values of a and b produce stable oscillations. Find the boundary between oscillations and equilibrium.