MATH 131 COMPUTER LAB 2: NUMERICAL METHODS

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Please type your report, but if you have trouble typing mathematical notation, feel free to write in the math neatly by hand. You can use any program you like for your calculations. If you've never programmed before, I recommend Maple as it has the easiest learning curve and you can build on my demos from lectures. You can also type paragraphs of text directly into your worksheet. If you are new to programming in Maple, you might find it easiest to set up one worksheet and make a copy for each different differential equation. That way you can modify one copy and your changes won't interfere with your calculations for another problem. If you do this, please print out a complete worksheet for each equation, and remember to restart Maple before running each worksheet. To restart Maple, push the restart button on the tool bar, or evaluate the command `restart;`

If you're more comfortable with Maple and procedures and such, feel free to write a more elaborate worksheet that can do multiple calculations without the need for multiple worksheets.

1. Euler's Method

Begin by implementing Euler's method. (Follow the book, p. 120–121.) Test that your worksheet is correct by reproducing the numbers from figure 2.4.4 on p. 113. You don't have to print out this check.

Deliverable #1: Consider the initial value problem

$$\frac{dy}{dx} = \frac{4}{1+x^2}, \quad y(0) = 0. \tag{1}$$

Solve this equation symbolically, and show that $y(1) = \pi$.

Deliverable #2: Using Euler's method and Equation (1), compute an approximation of π accurate to 3 decimal places: Specifically, start with 10 steps, and approximate $y(1)$. Double the number of steps until your approximations with n and $2n$ steps are within 0.0005 of each other. (That's a good rule of thumb to indicate when the number of steps is high enough.) Turn in a printout of your worksheet for the last approximation, and a table of approximations of π for each number of steps you tried and how much they differ from the actual value of π.

2. Improved Euler Method

Follow the book (p. 130–131) and implement the improved Euler method. To check your program, use it to reproduce the numbers from the fourth column in figure 2.5.4 on p. 126. You don't have to print out the results of this check.

Deliverable #3: Repeat Deliverable 2 but use the improved Euler method, and approximate π to 5 decimal places instead of 3. That is, look for approximations with n and $2n$ steps that are within 5×10^{-6} of each other.

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3. Logistic Population with Periodic Harvesting

(This part is based on the application at the end of Section 2.5, p. 132.)

Consider this logistic equation with periodic harvesting:

$$\frac{dy}{dt} = ky(M - y) - h \sin\left(\frac{2\pi t}{P}\right) \tag{2}$$

Here, y is the population size and P is the period of the harvesting. (Imagine a lake, for example, where most of the harvesting is from fishing during the summer tourist season, and not much happens in winter when it freezes over.)

In what follows, set $k = 1/2$, $M = 7$, $P = 1$, and harvesting rate $h = 2$. You have to be careful with variable names here. The equation uses h for harvesting rate, and you don't want to confuse this with the step size. You might want to use `H` or `hRate` in your Maple worksheet to avoid collision with `h`.

Note: To get π in Maple, you need to type `Pi`. If you type `pi` instead, Maple displays the same symbol π but treats that as a Greek letter that can be assigned to just like α, β, etc. This is a horrible point of confusion, and it can cause your worksheet to use up all the computer's memory and crash: If you accidentally use `pi`, Maple will do all the calculations with an unknown variable that looks like π, and since Maple can't evaluate it, it just keeps creating larger and larger symbolic expressions until your computer runs out of memory and crashes.

Deliverable #4: Use your improved Euler program to investigate Equation (2). Find a step size that gives reasonable results. Plot one solution that converges to a stable oscillation from above, one that converges from below, and one that goes off to negative infinity (extinction).

Deliverable #5: Look for a threshold y_* such that the population goes extinct if $y(0) < y_*$, but it oscillates stably if $y(0) > y_*$. Determine y_* down to two decimal places or so, and turn in plots that support your conclusion. Write a paragraph explaining how you found the threshold.

4. Stiff Equations

Consider this initial value problem:

$$\frac{dy}{dx} = -\frac{e^{x-1} + (x-1)\,y}{1 - x - 0.0001\,y}, \quad y(0) = \frac{1}{e} \tag{3}$$

Using the improved Euler method, try to approximate $y(1)$ to 2 decimal places. Start with 100 steps and double the number of steps, until you get the same answer to within 0.005 for n and $2n$ steps. It should become clear that something makes this problem more difficult than finding π. We're doing tons of calculations and getting only a couple of decimal places for our trouble. This is called a *stiff* equation, caused by the fact that there's that small number in an unfortunate place on the right hand side.

Deliverable #6: Write out a table of the numbers of steps and the approximation each gives. How close is each approximation to the correct answer (around -6.4977)?

Deliverable #7: As x approaches 1, what happens to the right hand side of Equation (3)? What happens to the steps taken by the numerical method as x approaches 1? Why might this cause trouble for numerical methods?

5. When Singularities Attack

Consider this initial value problem:

$$\frac{dy}{dx} = y + y^2, \quad y(0) = 1 \tag{4}$$

Deliverable #8: Using the improved Euler method, try to approximate $y(1)$. You'll see that something goes horribly wrong. Show some calculations and plots that illustrate the problem. Write a paragraph explaining what goes wrong and why. (*Hint:* You can actually solve this initial value problem symbolically...)