Simulating the Evolution of Regulatory Networks

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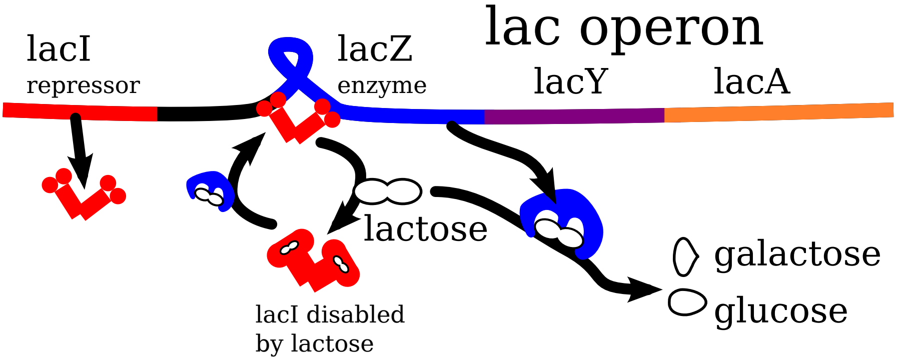
1. Regulatory networks
2. Simulated evolution
3. Rare events

Regulatory networks



Artificial life is difficult

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So I made a toy

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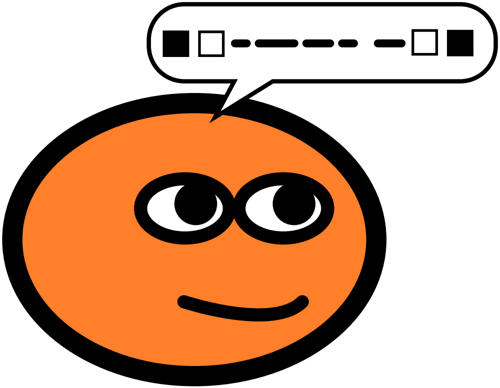
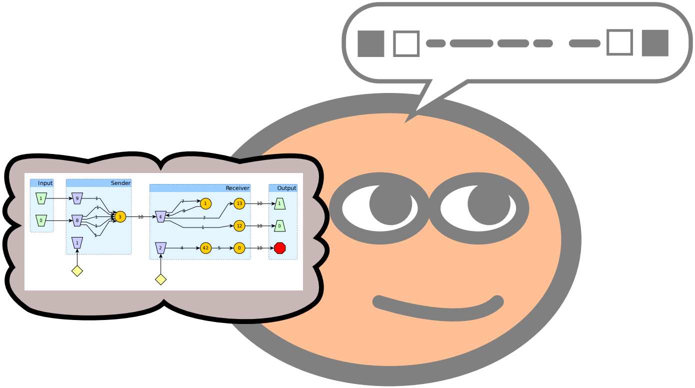
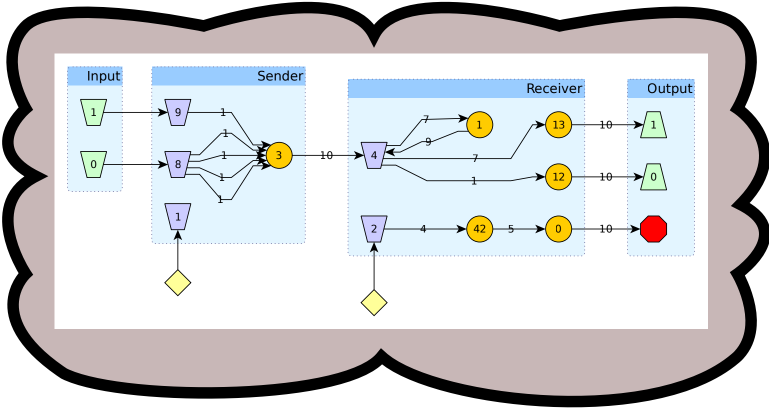
Agents, their job, & their brains



Agents, their job, & their brains

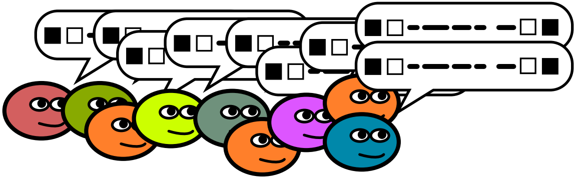


Agents, their job, & their brains

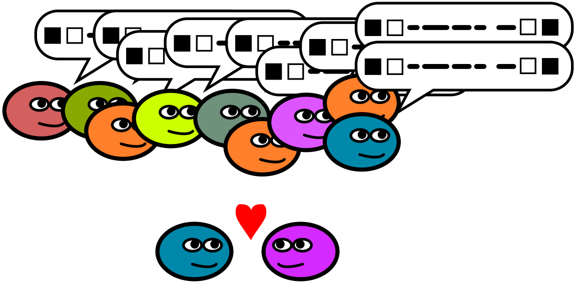


$A[8] \geq 1 \Rightarrow \operatorname{inc} A[3], \operatorname{dec} A[5]$

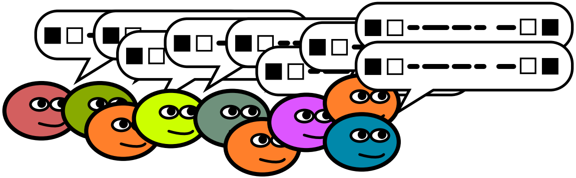
Selection-mutation processes

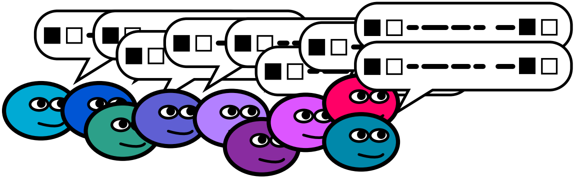


Selection-mutation processes

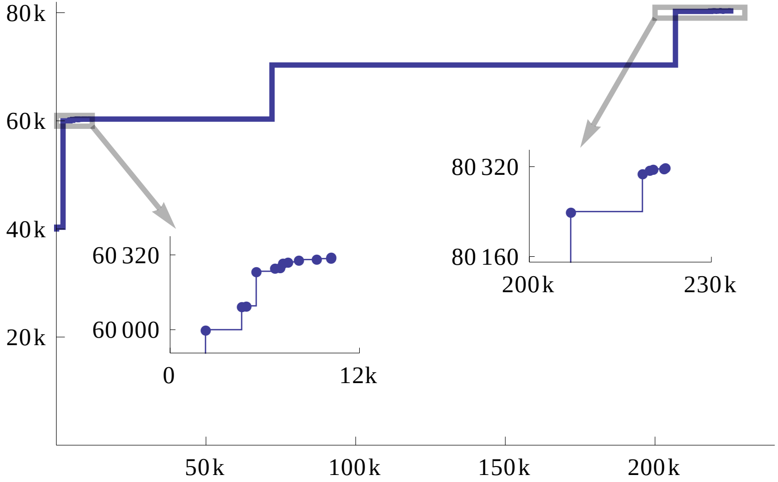


Selection-mutation processes

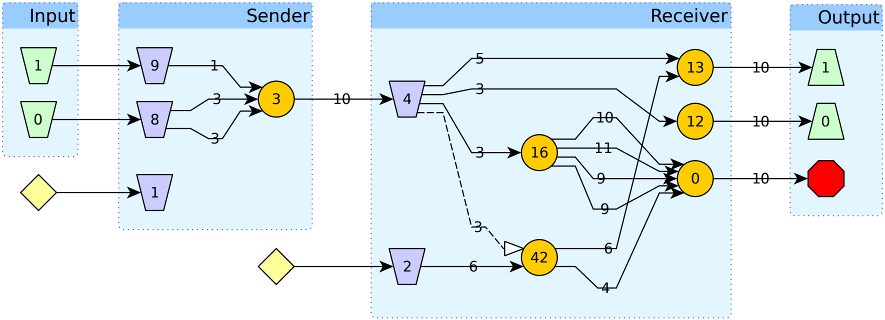




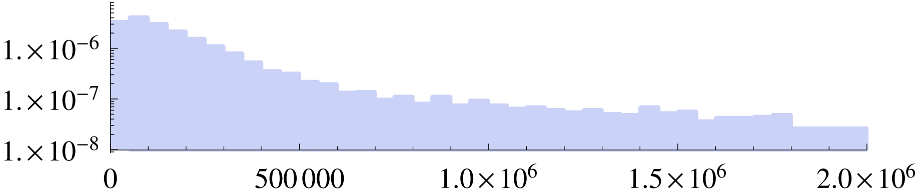
Punctuated equilibrium



Perfect solution



Time for last innovation



Waiting for a rare event

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Waiting for a rare event

- Time steps $t = 0, 1, 2, \ldots$
- All independent
- Waiting for a rare event
- $q =$ probability that it happens each time step
- q is small, think 1/100
- When does it first happen?

Waiting for a rare event

- $\mathbf{P}\,(\text{happens at } \textit{this} \text{ step}) = q$

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- $\mathbf{P}\left(\text{happens at } \textit{this} \text{ step}\right) = q$
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  - $\mathbf{P}\left(\text{first happens at } t = 1\right) = (1 - q)q$

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- $\mathbf{P}$(happens at *this* step) $= q$
- $\mathbf{P}$(first happens at $t = 0$) $= q$
- $\mathbf{P}$(first happens at $t = 1$) $= (1 - q)q$
- $\mathbf{P}$(first happens at $t = 2$) $=?$

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  - $\mathbf{P}(\text{first happens at } t = 2) = (1 - q)^2 q$

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- ...

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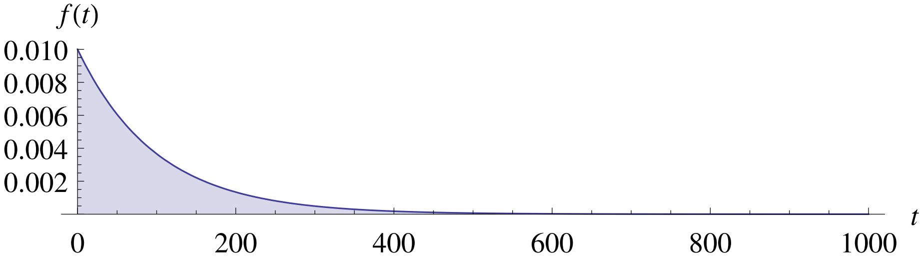
- $\mathbf{P}\,$(first happens at $t = 1$) $= (1 - q)q$
- $\mathbf{P}\,$(first happens at $t = 2$) $= (1 - q)^2 q$
- $\ldots$
- $\mathbf{P}\,$(first happens at t) $= (1 - q)^t q$
  - Doesn't happen on steps $0, 1, \ldots t - 1$
  - Does happen on step t

Geometric distribution

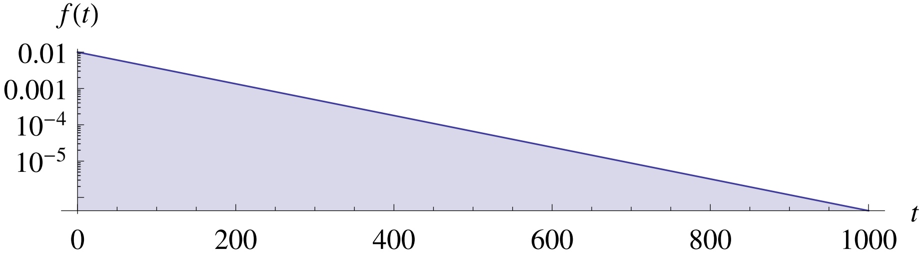
$$f(t) = \mathbf{P}(\text{first happens at } t)$$
$$f(t) = (1-q)^t q$$



Geometric distribution

$$f(t) = (1-q)^t q$$

$$\ln\left(f(t)\right) = \ln\left((1-q)^t q\right) = t\ln(1-q) + \ln(q)$$



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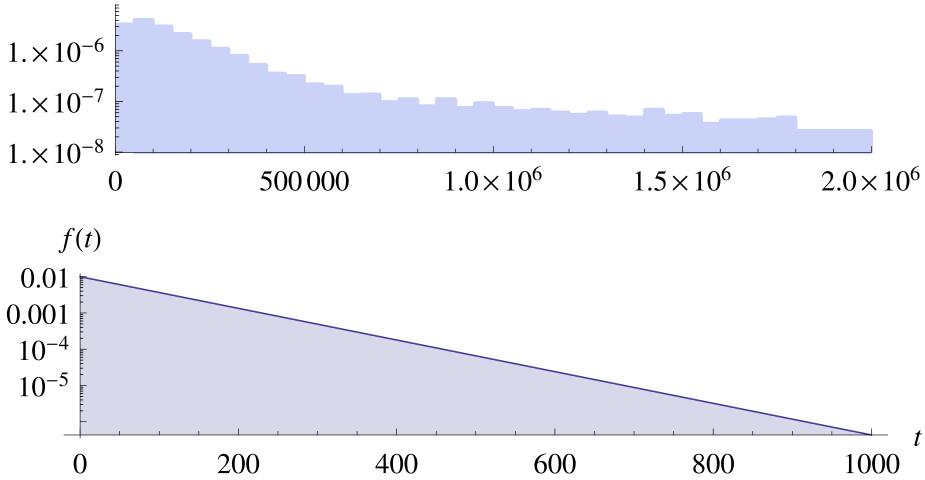

Conclusion

- Distribution of time for last innovation *isn't* geometric
- Biased toward smaller times
- More likely to happen shortly after next-to-last innovation
- Some kind of memory effect?

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